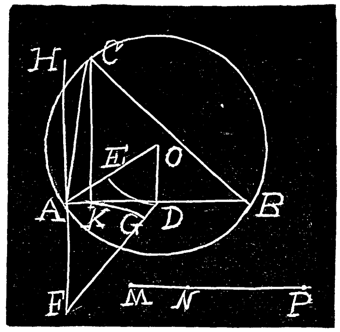
GEOMETRY.

379. **Proposed by G. I. HOPKINS, Manchester, N. H.**

Construct the triangle, having given base, vertical angle, and ratio of its altitude to difference of other two sides.

Solution by A. H. HOLMES, Brunswick, Maine.

Describe a circle, center O, in which the chord AB (=given base) will subtend the given angle. Draw OD perpendicular to AB, and draw OB. With OD as a radius and O as a center, describe an arc cutting OB in E. Let the divisions MN and NP of the line MP represent the ratio of the difference of the sides of the triangle to the perpendicular. Construct the fourth proportional to MN, NP, and BE, and erect this line, BF, perpendicular to AB at the point B. Draw DF. With F as a center, and FB as a radius, describe an arc cutting FD in G. Find fourth proportional to MN, NP, and $2DG$. Erect this line, BH, perpendicular to AB at point B. Draw HC parallel to AB and cutting circumference in C. Draw AC and BC. Then ABC will be the required triangle, which is shown as follows: $AB=2a$; r, the radius, $=\dfrac{a}{\cos C}$. Put $x+y$ and $x-y$ for the sides, and p for the ratio of difference of sides to perpendicular. Then we have the proportion, $2r:x+y=x-y:2py$.



$\therefore x^2=y^2+2pry\ldots(1)$.

Also, $\cos C=\dfrac{(x+y)^2+(x-y)^2-4a^2}{2(x+y)(x-y)}$.

$\therefore (1-\cos C)x^2+(1+\cos C)y^2=2a^2\ldots(2)$.

Eliminating x^2 from (1) and (2), $y^2+2pr(1-\cos C)y=a^2$.

$\therefore y=[a^2+p^2r^2(1-\cos C)^2]-pr(1-\cos C)$.

$\therefore$ Perpendicular $CK=2py=2p\{\sqrt{[a^2+p^2r^2(1-\cos C)^2]}-pr(1-\cos C)\}$.

Also solved by V. M. Spunar.

380. **Proposed by W. J. GREENSTREET, A. M., Stroud, England.**

$A\ B\ C\ D$ is a quadrilateral, sides in order a, b, c, d, and $B+D=\theta$. Express the diagonals in terms of a, b, c, d, θ.

Solution by A. H. HOLMES, Brunswick, Maine.

Let $ABCD$ be the rhombus, in which $AB=a$, $BC=b$, $CD=c$, $DA=d$. Put $x=AC$, $y=BD$, and $\angle ABC=\phi$. Then $\angle ADC=\theta-\phi$.

In $\triangle ABC$ we have, $\cos\phi=\dfrac{a^2+b^2-x^2}{2ab}$, and in $\triangle ADC$, $\cos(\theta-\phi)=$